# AFRL-RY-WP-TP-2009-1303

# EFFECT OF BANDWIDTH ON WIDEBAND-STAP PERFORMANCE (BRIEFING CHARTS)

**Ke Yong Li, S. Unnikrishna Pillai, and Joseph R. Guerci**

**C & P Technologies, Inc.**

**OCTOBER 2007**

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**SENSORS DIRECTORATE**
**WRIGHT-PATTERSON AIR FORCE BASE, OH 45433-7320**
**AIR FORCE MATERIEL COMMAND**
**UNITED STATES AIR FORCE**

| REPORT DOCUMENTATION PAGE | Form Approved OMB No. 0704-0188 |
|---|---|



| **1. REPORT DATE *(DD-MM-YY)*** | **2. REPORT TYPE** | **3. DATES COVERED *(From - To)*** |
|---|---|---|
| October 2007 | Conference Briefing Charts | 25 May 2006 – 19 October 2007 |

**4. TITLE AND SUBTITLE**
EFFECT OF BANDWIDTH ON WIDEBAND-STAP PERFORMANCE (BRIEFING CHARTS)

**5a. CONTRACT NUMBER** FA8750-06-C-0117

**5b. GRANT NUMBER**

**5c. PROGRAM ELEMENT NUMBER** 62204F

**6. AUTHOR(S)**
Ke Yong Li (C & P Technologies, Inc.)
S. Unnikrishna Pillai (Polytechnic University)
Joseph R. Guerci (Guerci Consulting)

**5d. PROJECT NUMBER** 5017

**5e. TASK NUMBER** RL

**5f. WORK UNIT NUMBER** 517R1511

**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**
C & P Technologies, Inc.
317 Harrington Avenue
Suites 9 & 10
Closter, NJ 07624-1911

Polytechnic University
Brooklyn, NY 11201

Guerci Consulting
2209 North Utah Street
Arlington, VA 22207

**8. PERFORMING ORGANIZATION REPORT NUMBER**

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**
Air Force Research Laboratory
Sensors Directorate
Wright-Patterson Air Force Base, OH 45433-7320
Air Force Materiel Command
United States Air Force

**10. SPONSORING/MONITORING AGENCY ACRONYM(S)** AFRL/RYRT

**11. SPONSORING/MONITORING AGENCY REPORT NUMBER(S)** AFRL-RY-WP-TP-2009-1303

**12. DISTRIBUTION/AVAILABILITY STATEMENT**
Approved for public release; distribution unlimited.

**13. SUPPLEMENTARY NOTES**
Conference presentation published in the Proceedings of the 41st Annual Asilomar Conference on Signals, Systems, and Computers, held November 04 - 07, 2007 at the Asilomar Hotel and Conference Grounds, Pacific Grove, CA. PAO Case Number: WPAFB 07-0327; Clearance date: 05 Nov 2007. Briefing contains color. See also, AFRL-RY-WP-TP-2009-1304 for a preprint version of the paper based on the briefing, and AFRL-RY-WP-TP-2009-1305 for a postprint version.

**14. ABSTRACT**
A wideband signal occupies a finite bandwidth that is significant compared to its carrier frequency. As a result, when transmitted, its returns cause bandwidth dispersion across the antenna. It is shown here that the effect of the finite bandwidth is to introduce a set of uncorrelated return signals for every physical scatter in the field. Further, each such uncorrelated return contains a set of coherent signals with different directional and Doppler components that result from a jittering effect both in angle and Doppler domain. As a result, adaptive clutter cancellation using traditional processing schemes does not work well. Although in principle it is possible to correct these decorrelating effects by 3D spacetime adaptive processing (STAP), the present day methods are quite costly and difficult to implement. In addition to the new wide band signal modeling framework mentioned above, we outline a hierarchical processing scheme which has the potential for dramatically reducing both processing and sample support burdens.

**15. SUBJECT TERMS**
wideband space-time adaptive processing

| **16. SECURITY CLASSIFICATION OF:** | | | **17. LIMITATION OF ABSTRACT:** | **18. NUMBER OF PAGES** | **19a. NAME OF RESPONSIBLE PERSON** (Monitor) |
|---|---|---|---|---|---|
| **a. REPORT** | **b. ABSTRACT** | **c. THIS PAGE** | | | Michael J. Callahan |
| Unclassified | Unclassified | Unclassified | SAR | 24 | **19b. TELEPHONE NUMBER** *(Include Area Code)* N/A |

**Standard Form 298 (Rev. 8-98)**
Prescribed by ANSI Std. Z39-18

# Effect of Bandwidth on Wideband-STAP Performance

**Unnikrishna S. Pillai**
**Polytechnic University, Brooklyn, NY**

**Ke Yong Li**
**C & P Technologies, Inc., Closter, NJ**

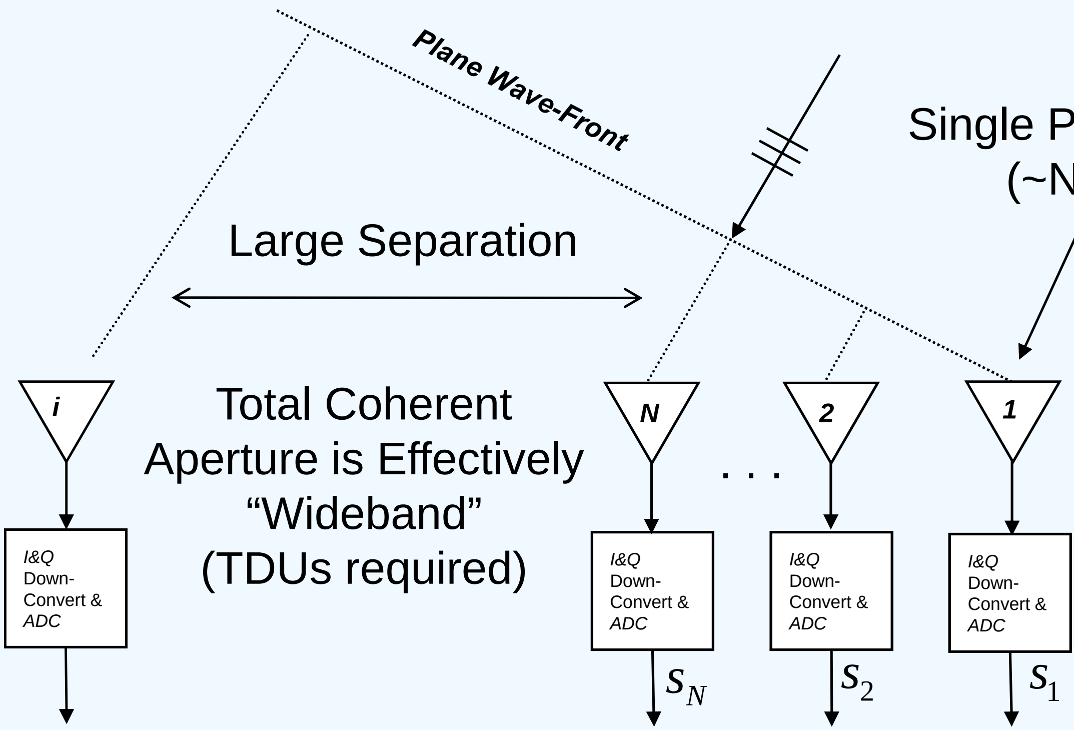
**Joseph R. Guerci**
**Guerci Consulting, Arlington, VA**

# Outline

- Wideband array
- Frequency sensitive gain pattern
- Effect of wideband on STAP
- Wideband data model – A new theory
- Wideband and coherency
- Conclusions

# Wideband

**Large separation between multiple transmitters and/or receivers results in significant antenna dispersion even for modest bandwidths!**




Wideband Condition:

$$\frac{\text{Array Length}}{c} > \frac{1}{BW}$$

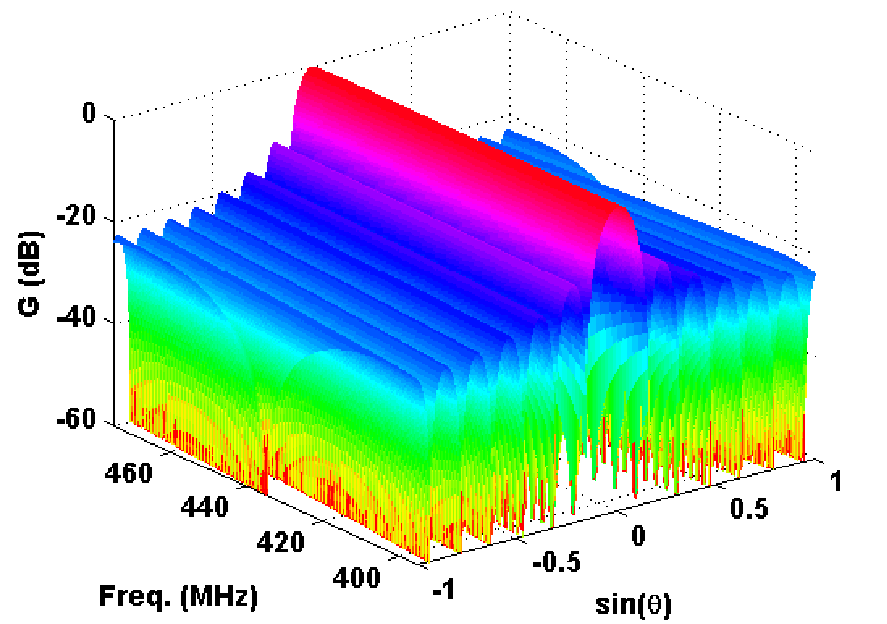
# Frequency Sensitive Array Gain Pattern

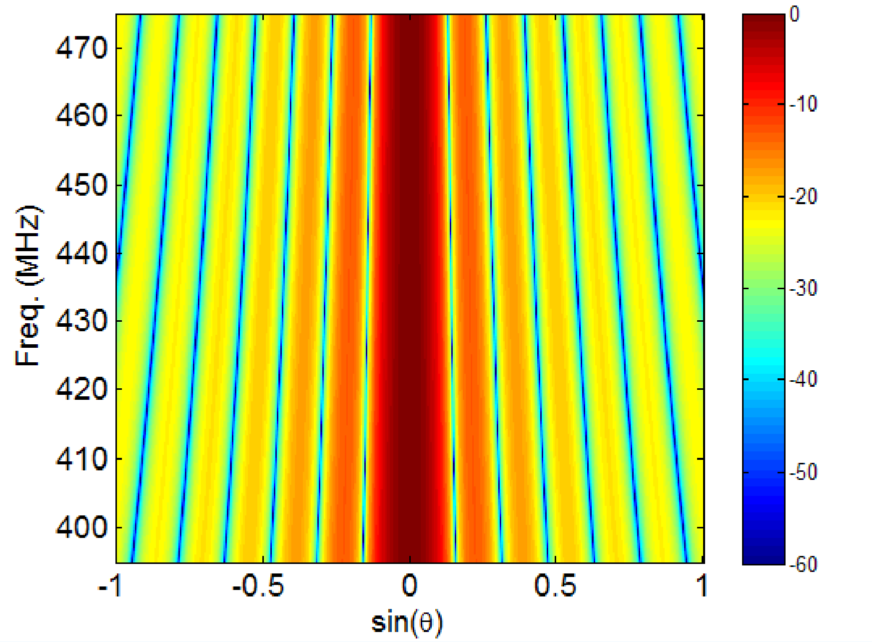
## Array Amplitude Pattern

$$C(\theta,\,\omega_k) = \sum_{i=1}^{N} e^{-j2\pi\frac{d}{\lambda_k}(i-1)\sin\theta},$$

## Array Gain Pattern

$$G(\theta,\,\omega_k) = \left|C(\theta,\,\omega_k)\right|^2$$




**Bandwidth = 395 MHz - 475 MHz (80 MHz), Sensors used: 14**

# Space-Time Data Vector

$N$ **sensors,** $M$ **pulses**

**Data Vector** $$\mathbf{x}(t) = \sum_k \alpha_k \mathbf{s}(\theta_k, \omega_{d_k}) + \mathbf{n}(t)$$

$\alpha_k$ : **Random scatter return,** $\mathbf{n}(t)$ : **Noise vector**

$\mathbf{s}(\theta_k, \omega_{d_k})$ : $MN \times 1$ **space-time steering vector**

$$\mathbf{s}_k = \mathbf{s}(\theta_k, \omega_{d_k}) = \underline{b}(\omega_{d_k}) \otimes \underline{a}(\theta_k)$$

**Spatial steering vector**

$$\underline{a}(\theta_k) = \left[1 \quad e^{-j\omega_o \frac{d\sin\theta_k}{c}} \quad e^{-j\omega_o \frac{2d\sin\theta_k}{c}} \quad \cdots \quad e^{-j\omega_o \frac{(N-1)d\sin\theta_k}{c}}\right]^T$$

**Temporal steering vector**

$$\underline{b}(\omega_{d_k}) = \left[1 \quad e^{-j\pi\omega_{d_k}} \quad e^{-j2\pi\omega_{d_k}} \quad \cdots \quad e^{-j(M-1)\pi\omega_{d_k}}\right]^T$$

# Space-Time Data Vector

**Doppler frequency** $\omega_{d_k} = \dfrac{2VT\sin\theta_k}{\lambda/2}$

$V$ : **Platform velocity**

$T$ : **Pulse repetition interval**

$\lambda$ : **Operating wavelength**

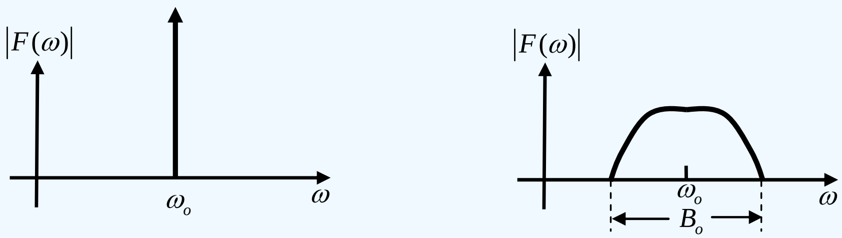
**The covariance matrix in an uncorrelated clutter and noise scene**

$$\mathbf{R}_x = \sum P_k \mathbf{s}_k \mathbf{s}_k^* + \sigma^2 \mathbf{I}$$

# Wideband Space-Time Data Vector

## Wideband transmit waveform:

$$f(t) \leftrightarrow F(\omega) = \begin{cases} F(\omega), & |\omega| \le B_o/2 \\ 0, & |\omega| > B_o/2 \end{cases}.$$




(a) Narrowband (b) Wideband

**Let** $y_1^{(1)}(t) = f(t)e^{j\omega_o t}$ **: scattered return at the reference sensor due to first pulse.**

$i^{th}$ **sensor:** $y_1^{(i)}(t) = f\left(t-(i-1)\tau_1\right)e^{j\omega_o\left(t-(i-1)\tau_1\right)},\ \ i=1,\ 2,\ \cdots\ N$

$\tau_1 = \dfrac{d\sin\theta_k}{c}$: **Interelement time delay for azimuth angle** $\theta_k$

# Wideband Space-Time Data Vector

**The first pulse output:**

$$\underline{y}_1(t) = \left[\, f(t)e^{j\omega_o t}, \;\; \cdots \; f\left(t-(N-1)\tau_1\right)e^{j\omega_o\left(t-(N-1)\tau_1\right)} \right]^T.$$

**Reference sensor due to $n^{th}$ pulse:**

$$y_n^{(1)}(t) = f\left(t-(n-1)\tau_2\right)e^{-j\pi(n-1)\omega_{d\,k}}\; e^{j\omega_o t}$$

$\tau_2 = \dfrac{2VT\sin\theta_k}{c}$: **Interpulse time delay for azimuth angle** $\theta_k$

**$i^{th}$ sensor output due to $n^{th}$ pulse:**

$$z_n^{(i)}(t) = f\left(t-(n-1)\tau_2-(i-1)\tau_1\right)e^{-j\pi(n-1)\omega_{d\,k}}\; e^{-j(i-1)\tau_1},$$

$$i=1,\,2,\,\cdots N, \quad n=1,\,2,\,\cdots\, M.$$

# Wideband Space-Time Data Vector

**Output vector for the $n^{th}$ pulse:** $\underline{z}_n(t) = \left[ z_n^{(1)}(t),\ z_n^{(2)}(t),\ \cdots\ z_n^{(N)}(t) \right]^T$

**Space-time vector:** $$\mathbf{z}(t) = \begin{bmatrix} \underline{z}_1(t) \\ \underline{z}_2(t) \\ \vdots \\ \underline{z}_M(t) \end{bmatrix} = \mathbf{f}_k(t) \circ \mathbf{s}(\theta_k,\ \omega_{d_k})$$

**Schur-Hadamard product**

$\mathbf{f}_k(t)$: $MN \times 1$ **transmit signal dependent vector whose $(iN+n)^{th}$ element is given by**

$$f\left(t - n\,\tau_2 - i\,\tau_1\right), \quad i = 0,\ 1,\ 2,\ \cdots N-1,$$
$$n = 0,\ 1,\ 2,\ \cdots, M-1\,.$$

# Wideband Clutter Covariance Matrix

**Covariance matrix for the wideband return** $\mathbf{f}_k(t)$

$$\mathbf{T}_k(\tau) = E\left\{\mathbf{f}_k(t)\mathbf{f}_k^*(t+\tau)\right\} > 0$$

**Wideband array output covariance matrix:**

$$\mathbf{R}_z = E\left\{\mathbf{z}(t)\mathbf{z}^*(t)\right\} = \sum_k P_k \mathbf{T}_k \circ \mathbf{s}_k \mathbf{s}_k^* + \sigma^2 \mathbf{I}$$

**Low pass transmit signal gives**

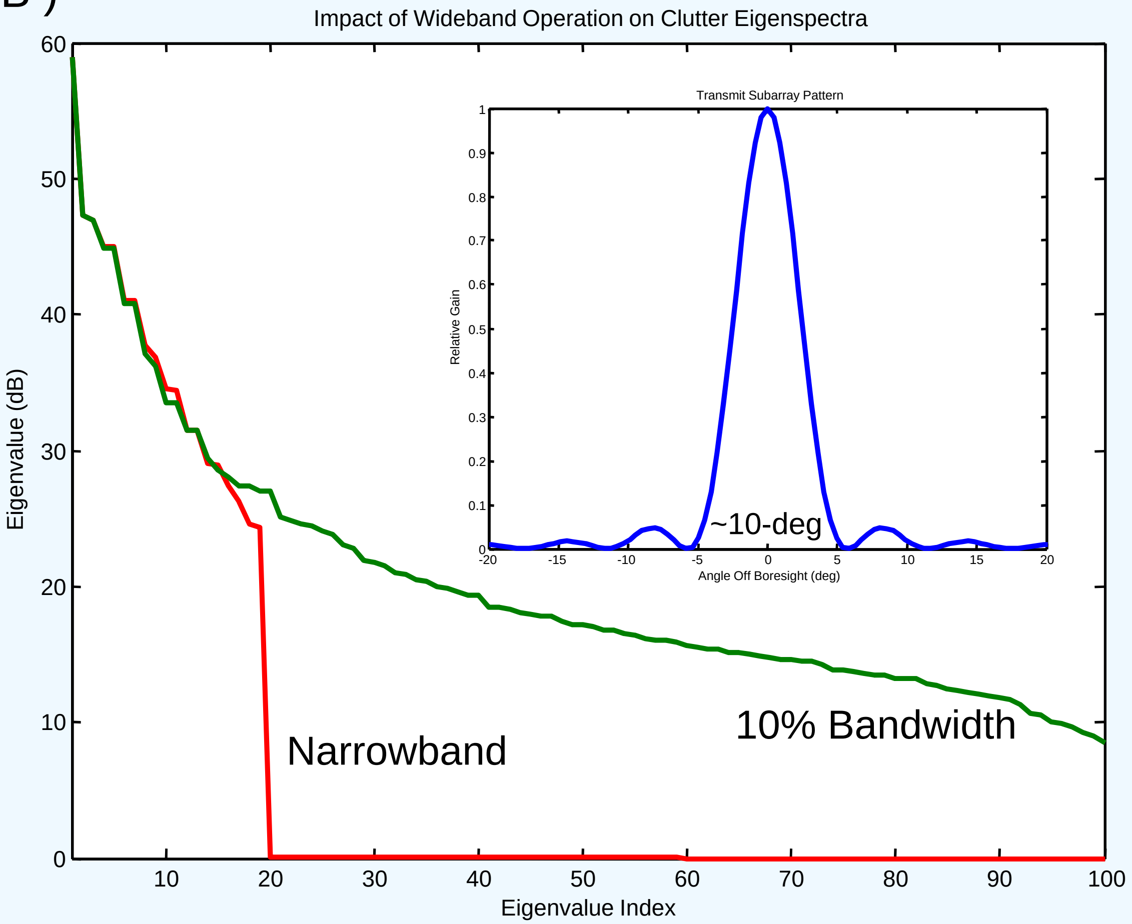
$$\mathbf{T}_k(m,\ n) = \operatorname{sinc} B_o\left((n_1 - n_2)\tau_2(k) + (i_1 - i_2)\tau_1(k)\right)$$

$$m = i_1 N + n_1, \quad n = i_2 N + n_2\ .$$

**If** $\mathbf{T}_k = \mathbf{T}$ **, then** $\mathbf{R}_z = \mathbf{T} \circ \mathbf{R}_x$

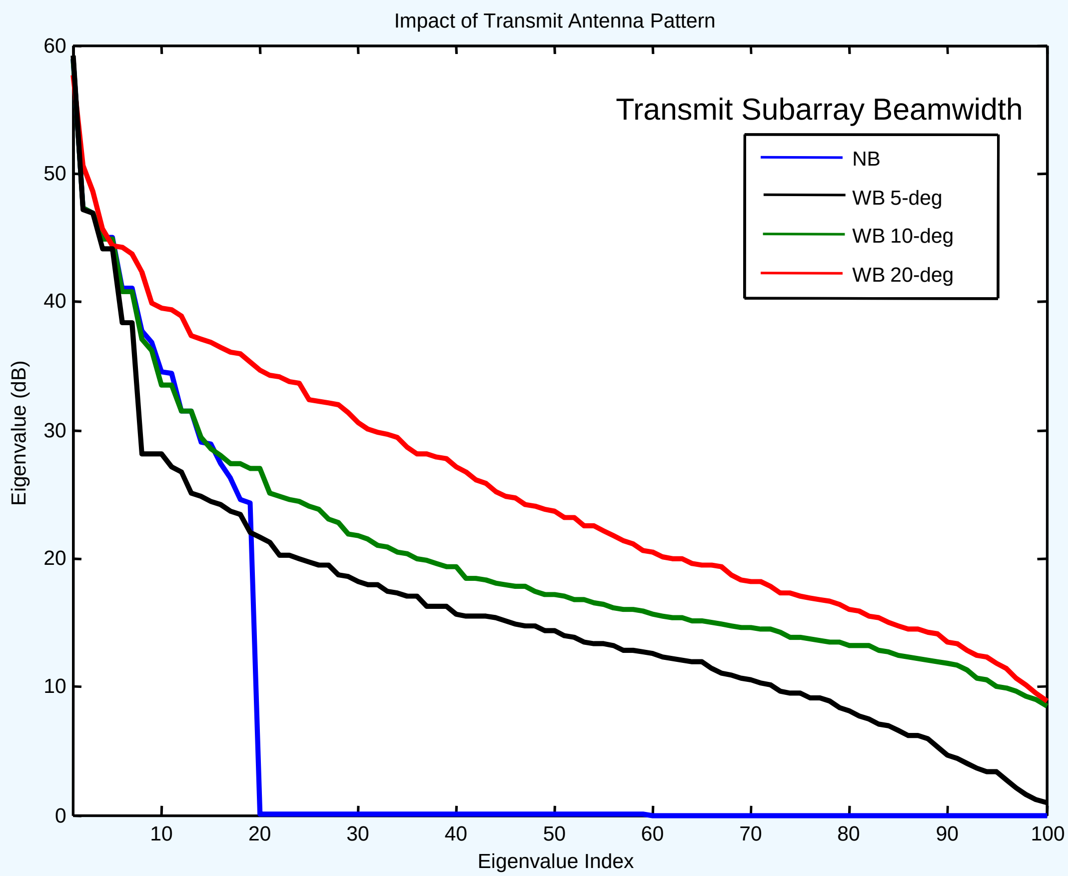
# Impact of Wideband on STAP

- **X-band example**
  - ( X-band case, $f_0$ = 10 GHz, $L$ = 3 m, $B$ = 10%, $N$ = 10 subarrays, CNR = 40 dB )



# Impact of Wideband on STAP

- **Transmitter plays a big role in shaping clutter characteristics**



# Impact of Wideband on STAP

**Narrowband clutter covariance matrix (single scatter at $\theta_o$)**

$$\mathbf{R}_x = P_o\,\mathbf{s}(\theta_o,\omega_{d_o})\mathbf{s}^*(\theta_o,\omega_{d_o})$$

**Wideband clutter covariance matrix:**

$$\mathbf{R}_z = \mathbf{T}_o \circ \left(P_o\,\mathbf{s}(\theta_o,\omega_{d_o})\mathbf{s}^*(\theta_o,\omega_{d_o})\right).$$

$\mathbf{T}_o = \sum_{i=1}^{MN} \lambda_i\,\mathbf{e}_i\,\mathbf{e}_i^* \geq 0$ **gives**

$$\mathbf{R}_z = \sum_{k=1}^{MN} P_k\,\tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^* + \sigma^2\mathbf{I}, \qquad P_k = P_o\lambda_k > 0, \qquad \tilde{\mathbf{s}}_k = \mathbf{e}_k \circ \mathbf{s}(\theta_o,\omega_{d_o}).$$

**A bunch of $MN$ uncorrelated returns, all of them associated with the single scatter located along $\theta_o$ .**

$\tilde{\mathbf{s}}_k$ **: Amplitude modulated steering vector associated with Doppler frequency $\omega_{d_o}$ and location $\theta_o$**

# Impact of Wideband on STAP

Let $\mathbf{d}_k$ represents the ordinary DFT vector associated with the eigenvector $\mathbf{e}_k$ . The entries in $\mathbf{e}_k$ corresponds to a double sampling period of $\tau_1$ followed by $\tau_2$ . Thus

$$\mathbf{e}_k = \sum_{n=1}^{MN} d_k(n) \left( \underline{b}\left(\frac{2\, j_n}{M}\right) \otimes \underline{a}\left(\frac{i_n\, c_o}{N}\right)\right)$$

Modified steering vector:

$$\tilde{\mathbf{s}}_k = \sum_{n=1}^{MN} d_k(n) \left\{ \underline{b}\left(\frac{2\, j_n}{M}\right) \otimes \underline{a}\left(\frac{i_n\, c_o}{N}\right)\right\} \circ \left( \underline{b}\left(\omega_{d_o}\right) \otimes \underline{a}\left(\theta_o\right)\right)$$

$$= \sum_{n=1}^{MN} d_k(n)\, \mathbf{s}(\theta_o + \frac{i_n\, c_o}{N}, \omega_{d_o} + \frac{2\, j_n}{M}) = \sum_{n=1}^{MN} d_k(n)\, \mathbf{s}(\theta_n, \omega_{d_n})$$

Coherent sum

$$\theta_n = \theta_o + \frac{i_n\, c_o}{N}, \quad \omega_{d_n} = \omega_{d_o} + \frac{2\, j_n}{M}.$$

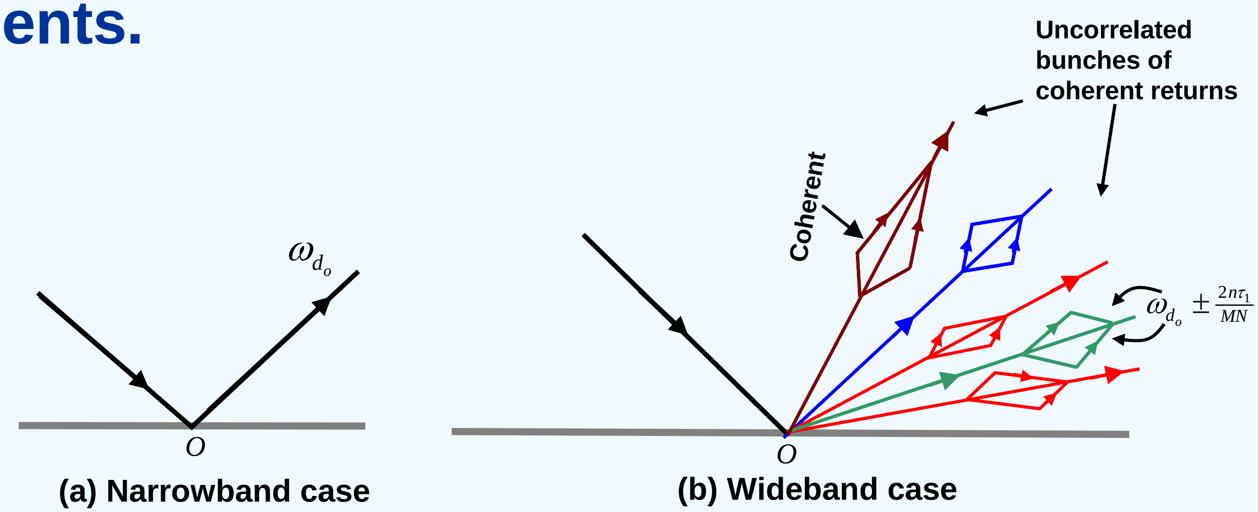
$\theta_n,\ n = 1 \rightarrow MN$

$\omega_{d_n},\ n = 1 \rightarrow MN$

# Impact of Wideband on STAP

Wideband introduces jittering effect both on angle and Doppler by generating a bunch of uncorrelated returns.

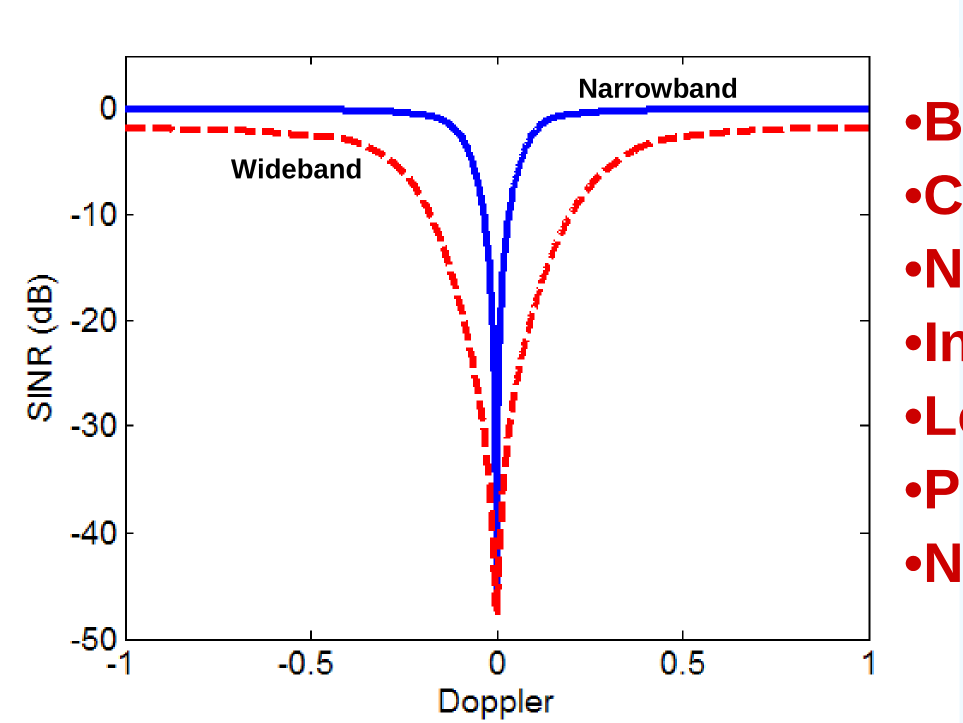
Each such uncorrelated return contains a set of coherent returns with different directional and Doppler components.




Modified steering vector $\tilde{\mathbf{s}}_k$ contains several coherent returns.

Adaptive processor will not be able to null out the clutter.

# SINR Loss With and Without Bandwidth Dispersion




- Bandwidth $BW = 80\,MHz$
- Center frequency $f_c = 435\,MHz$
- Number of sensors $N = 14$
- Interelement spacing $d = 0.33\,m$
- Look angle $\theta_o = 0^o$
- PRF = 625 Hz
- Number of pulse $M = 16$

# Conclusions

- Large aperture size contributes to wideband conditions
- Single scatter generates several uncorrelated return bunches.
- Each return bunch contains multiple coherent returns
- Adaptive processor generates wider null